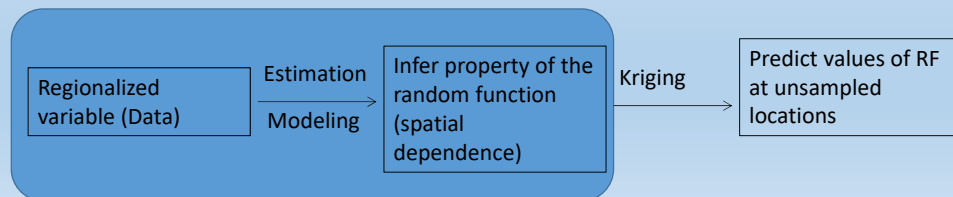


COA 616 Geostatistics in Environmental Sciences

Lecture 6 – More on Variograms and spatial interpolation

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Combining models

More complex functions to describe more complex variograms for large data set.

Combine two or more simple models. $\gamma(\vec{h}) = \sum_{i=1}^n w_i \gamma_i(\vec{h})$

Any combinations of CNSD function is CNSD.

Spatial dependence may occur at two distinct scales – represented in the variogram as two spatial components – nested structure.

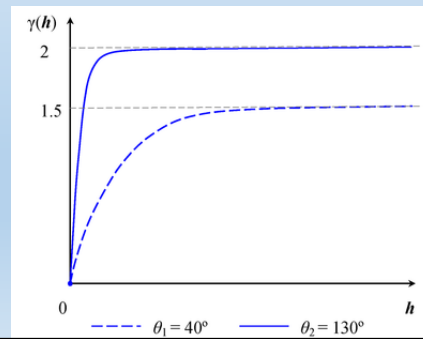
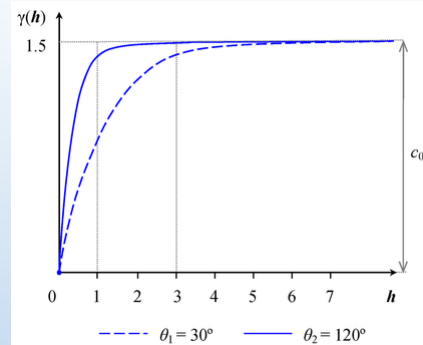
$$\gamma(\vec{h}) = \begin{cases} c_1 \left\{ \frac{3\vec{h}}{2a_1} - \frac{1}{2} \left(\frac{\vec{h}}{a_1} \right)^3 \right\} + c_2 \left\{ \frac{3\vec{h}}{2a_2} - \frac{1}{2} \left(\frac{\vec{h}}{a_2} \right)^3 \right\} & \text{for } 0 < \vec{h} < a_1 \\ c_1 + c_2 \left\{ \frac{3\vec{h}}{2a_2} - \frac{1}{2} \left(\frac{\vec{h}}{a_2} \right)^3 \right\} & \text{for } a_1 < \vec{h} < a_2 \\ c_1 + c_2 & \text{for } \vec{h} > a_2 \end{cases}$$

Anisotropy

Variation varies with direction

Geometric anisotropy: It occurs when the same sill (or scale) parameter is present in all directions but the range changes with direction.

Zonal anisotropy: the sill (or scale) parameter is different for different directions. It is not possible to transform such a structure into an isotropic semivariogram.



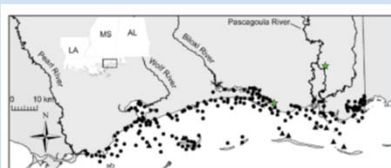
Spatial interpolation

Examples

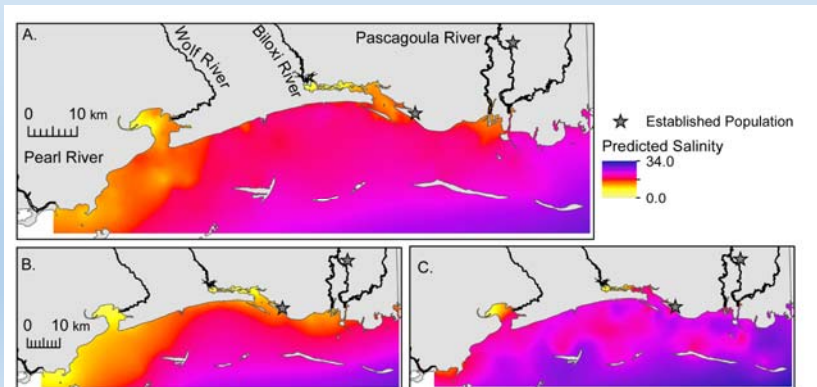
Predict rainfall from rain gauges

Predict salinity from limited measurements

Predict total nitrogen concentration in soil



Lowe et al. PLoS ONE 7(7)
E41580.



Spatial interpolation

Nearly all the methods of prediction can be seen as weighted averages of data

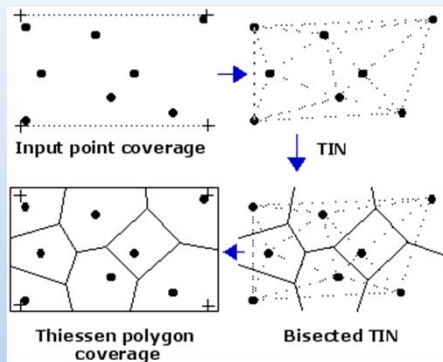
$$z^*(\vec{x}_0) = \sum_{i=1}^N \lambda_i z(\vec{x}_i)$$

where \vec{x}_0 is a target point for which we want an estimate

\vec{x}_i s are locations where we have measurements

λ_i are the weights

Spatial interpolation – Thiessen polygons (nearest neighbor)

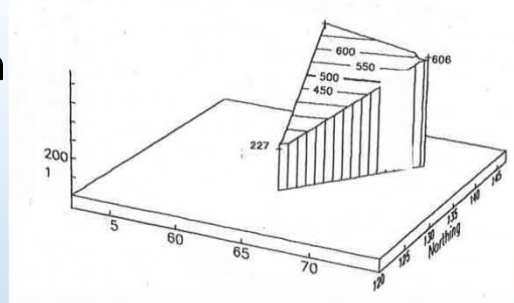


$$\lambda_i = \begin{cases} 1 & \text{if } \vec{x}_i \in V_i \\ 0 & \text{otherwise} \end{cases}$$

Each prediction is based on just one measurement, and information from neighboring points is ignored.

No estimate of the error.

Spatial interpolation – Triangulation



Fitting a plane through three samples that surround the point being estimated.

We can calculate the coefficients a , b , and c by solving the following system of equations :

$$ax_1 + by_1 + c = z_1$$

$$ax_2 + by_2 + c = z_2$$

$$ax_3 + by_3 + c = z_3$$

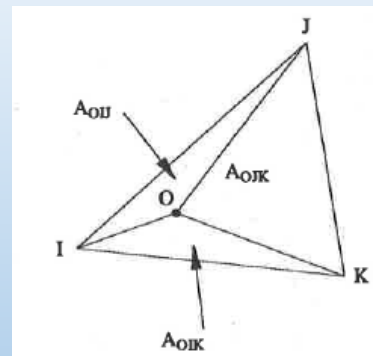
Then the estimate at (x_0, y_0) becomes : $ax_0 + by_0 + c$

Spatial interpolation – Triangulation II

We could also express our estimates as a weighted linear combination of the three sample values.

$$\hat{v}_o = \frac{A_{OJK}v_I + A_{OIK}v_J + A_{OIJ}v_K}{A_{IJK}}$$

where A s represent the areas of the triangles given in their subscripts.



Disadvantage

Prediction depends on only three data

No measure of error

No obvious triangulation that is better than any other (a **Delaunay triangulation** for a set \mathbf{P} of points in a plane is a triangulation $DT(\mathbf{P})$ such that no point in \mathbf{P} is inside the circumcircle of any triangle in $DT(\mathbf{P})$).

Spatial interpolation – Inverse functions of distance

$$\lambda_i = 1/|x_i - x_0|^\beta \quad \beta > 0$$

$$\sum \lambda_i = 1$$

The interpolation is exact, no discontinuity.

Disadvantage:

The choice of weighting function is arbitrary

No measured of error

Takes no account of configuration of the sampling

Spatial interpolation – Trend surfaces

A form of multiple regression in which the predictors are the spatial coordinates.

$$z(x_1, x_2) = f(x_1, x_2) + \varepsilon$$

Plausible functions, usually simple polynomials (1st, 2nd and 3rd order), are fitted by least squares to the spatial coordinates.

$$z = b_0 + b_1x_1 + b_2x_2$$

$$z = b_0 + b_1x_1 + b_2x_2 + b_3x_1^2 + b_4x_2^2 + b_5x_1x_2$$

Spatial variation is complex that a polynomial of very higher order is needed, and the results are unstable.

The residuals from the trend are autocorrelated.

It can represent long-range trend but not sufficiently local trend

Spatial interpolation – Splines

It consists of polynomials but each polynomial only describes pieces of a line or surface. They are fitted together so that they join smoothly.

