Lecture 9
Spatial Interpolation

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INTERPOLATION

Procedure to predict values of attributes at unsampled points

Why?
Can't measure all locations:
  Time
  Money
  Impossible (physical- legal)

Changing cell size
Missing/unsuitable data
Past date (eg. temperature)
### Systematic sampling pattern

- **Easy**
- Samples spaced uniformly at fixed X, Y intervals
- Parallel lines

**Advantages**
- Easy to understand

**Disadvantages**
- All receive same attention
- Difficult to stay on lines
- May be biases

### Random Sampling

- Select point based on random number process
- Plot on map
- Visit sample

**Advantages**
- Less biased *(unlikely to match pattern in landscape)*

**Disadvantages**
- Does nothing to distribute samples in areas of high concentration
- Difficult to explain, location of points may be a problem
Cluster Sampling

Cluster centers are established
(random or systematic)
Samples arranged around each center
Plot on map
Visit sample
(e.g. US Forest Service, Forest Inventory Analysis (FIA)
Clusters located at random then systematic pattern of samples at that location)

Advantages
Reduced travel time

Adaptive sampling

More sampling where there is more variability.
Need prior knowledge of variability, e.g. two stage sampling

Advantages
More efficient, homogeneous areas have few samples, better representation of variable areas.

Disadvantages
Need prior information on variability through space
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Many methods - All combine information about the sample coordinates with the magnitude of the measurement variable to estimate the variable of interest at the unmeasured location.

Methods differ in weighting and number of observations used.

Different methods produce different results.

No single method has been shown to be more accurate in every application.

Accuracy is judged by withheld sample points.

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Outputs typically:

Raster surface
- Values are measured at a set of sample points
- Raster layer boundaries and cell dimensions established
- Interpolation method estimate the value for the center of each unmeasured grid cell

Contour Lines
Iterative process
- From the sample points estimate points of a value. Connect these points to form a line
- Estimate the next value, creating another line with the restriction that lines of different values do not cross.
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1st Method - Thiessen Polygon

Assigns interpolated value equal to the value found at the nearest sample location

Conceptually simplest method

Only one point used (nearest)

Often called nearest sample or nearest neighbor
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Thiessen Polygon

Advantage:
Ease of application

Accuracy depends largely on sampling density

Boundaries often odd shaped as transitions between polygons are often abrupt

Continuous variables often not well represented

Thiessen Polygon

1. Draw lines connecting the points to their nearest neighbors.
2. Find the bisectors of each line.
3. Connect the bisectors of the lines and assign the resulting polygon the value of the center point
Sampled locations and values

Thiessen polygons
Natural neighbor interpolation

• Based on Thiessen Polygon Network

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Fixed-Radius – Local Averaging

More complex than nearest sample

Cell values estimated based on the average of nearby samples

Samples used depend on search radius

(any sample found inside the circle is used in average, outside ignored)

• Specify output raster grid
• Fixed-radius circle is centered over a raster cell

Circle radius typically equals several raster cell widths

(causes neighboring cell values to be similar)

Several sample points used

Some circles may contain no points

Search radius important; too large may smooth the data too much
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Fixed-Radius – Local Averaging

Input

Output

sample radius

observation

interpolation point

output location and values

fixed radius sample size

output value the average of three samples

output value based on one sample

Input sample layer

Interpolated surface
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**Fixed-Radius – Local Averaging**

![Original surface](image1.png) ![Fixed radius](image2.png)

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**Inverse Distance Weighted (IDW)**

Estimates the values at unknown points using the distance and values to nearby known points (*IDW reduces the contribution of a known point to the interpolated value*)

Weight of each sample point is an inverse proportion to the distance.

The further away the point, the less the weight in helping define the unsampled location.
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*Inverse Distance Weighted (IDW)*

Zi is value of known point

Dij is distance to known point

Zj is the unknown point

n is a user selected exponent

\[
Z_j = \frac{\sum \frac{Z_i}{d_{ij}^n}}{\sum \frac{1}{d_{ij}^n}}
\]

**Figure 9-7:** An example calculation for a linear inverse distance weighted interpolator. Known points are weighted by the inverse of their distance from the interpolation point, and summed. The sum is then divided by the
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Inverse Distance Weighted (IDW)

Factors affecting interpolated surface:

• Size of exponent, n affects the shape of the surface
  larger n means the closer points are more influential

• A larger number of sample points results in a smoother
  surface
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Inverse Distance Weighted (IDW)

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Trend Surface Interpolation

Fitting a statistical model, a trend surface, through the measured points. (typically polynomial)

\[ Z = a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 y^2 + a_5 x y \]

Where \( Z \) is the value at any point \( x \)
Where \( a_i \)s are coefficients estimated in a regression model
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Trend Surface Interpolation

Spline functions are used to interpolate along a smooth curve.

NAME derived from the drafting tool, a flexible ruler, that helps create smooth curves through several points.

Spline functions are used to interpolate along a smooth curve.

Force a smooth line to pass through a desired set of points.

Constructed from a set of joined polynomial functions.

Figure 9.9: Sample points and contours from the original surface (left), and sample points and derived contours from a third order trend surface fit to the sample points (right).
INTERPOLATION: Splines

Original surface  Spline interpolation

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Kriging

Similar to Inverse Distance Weighting (IDW)

Kriging uses the minimum variance method to calculate the weights rather than applying an arbitrary or less precise weighting scheme.
Interpolation

Kriging

Method relies on spatial autocorrelation

Higher autocorrelations, points near each other are alike.

**Figure 9-13:** Spatially autocorrelated (top) and spatially uncorrelated (bottom) data layers. Plots of nearby sample pairs (right panels) show similar values for autocorrelated Layer 1, and unrelated values for uncorrelated layer 2.

**INTERPOLATION** (cont.)

Exact/Non Exact methods

Exact – predicted values equal observed
Theissen
IDW
Spline

Non Exact-predicted values might not equal observed
Fixed-Radius
Trend surface
Interpolation in ArcGIS: Spatial Analyst