

Lecture 9
Spatial Interpolation

Wei Wu
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INTERPOLATION

Procedure to predict values of attributes at
unsampled points

Why?

Can't measure all locations:

Time

Money

Impossible (physical- legal)

Changing cell size

Missing/unsuitable data

Past date (*eg. temperature*)

Systematic sampling pattern

Easy
Samples spaced uniformly at
fixed X, Y intervals
Parallel lines

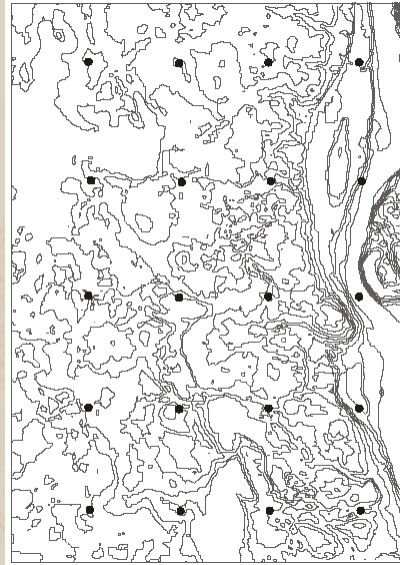
Advantages

Easy to understand

Disadvantages

All receive same attention
Difficult to stay on lines

May be biases



Random Sampling

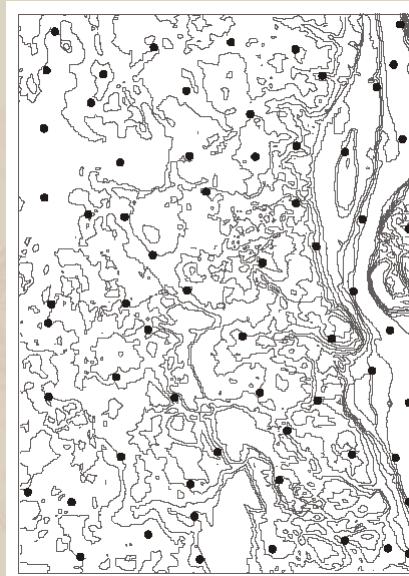
Select point based on
random number process
Plot on map
Visit sample

Advantages

Less biased (*unlikely to match
pattern in landscape*)

Disadvantages

Does nothing to distribute
samples in areas of high
Difficult to explain, location
of points may be a problem



Cluster Sampling

Cluster centers are established
(*random or systematic*)

Samples arranged around each
center

Plot on map

Visit sample

(*e.g. US Forest Service, Forest
Inventory Analysis (FIA)*)

*Clusters located at random then
systematic pattern of samples at
that location*)



Advantages

Reduced travel time

Adaptive sampling

More sampling where there is
more variability.

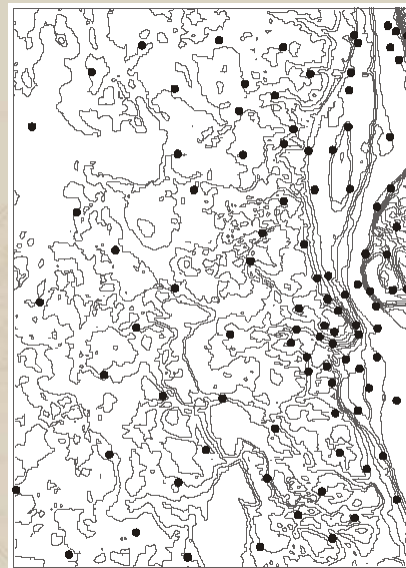
Need prior knowledge of
variability, e.g. two stage
sampling

Advantages

More efficient, homogeneous
areas have few samples,
better representation of
variable areas.

Disadvantages

Need prior information on
variability through space



INTERPOLATION

Many methods - All combine information about the sample coordinates with the magnitude of the measurement variable to estimate the variable of interest at the unmeasured location

Methods differ in weighting and number of observations used

Different methods produce different results

No single method has been shown to be more accurate in every application

Accuracy is judged by withheld sample points

INTERPOLATION

Outputs typically:

Raster surface

- *Values are measured at a set of sample points*
- *Raster layer boundaries and cell dimensions established*
- *Interpolation method estimate the value for the center of each unmeasured grid cell*

Contour Lines

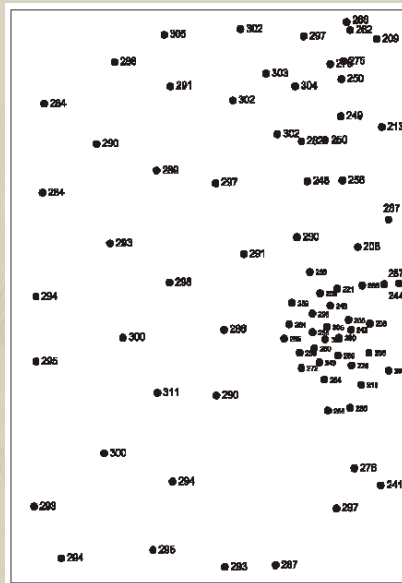
Iterative process

- *From the sample points estimate points of a value Connect these points to form a line*
- *Estimate the next value, creating another line with the restriction that lines of different values do not cross.*

Example Base



Elevation contours



Sampled locations and values

INTERPOLATION

1st Method - Thiessen Polygon

Assigns interpolated value equal to the value found at the nearest sample location

Conceptually simplest method

Only one point used (nearest)

Often called nearest sample or nearest neighbor

INTERPOLATION

Thiessen Polygon

Advantage:
Ease of application

Accuracy depends largely on sampling density

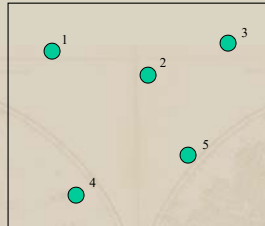
Boundaries often odd shaped as transitions between polygons are often abrupt

Continuous variables often not well represented

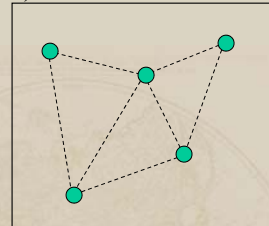
Thiessen Polygon

- Start:
1. Draw lines connecting the points to their nearest neighbors.
 2. Find the bisectors of each line.
 3. Connect the bisectors of the lines and assign the resulting polygon the value of the center point

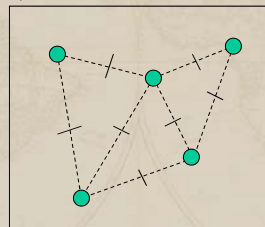
Start:



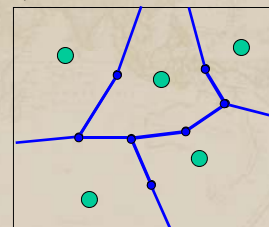
1)

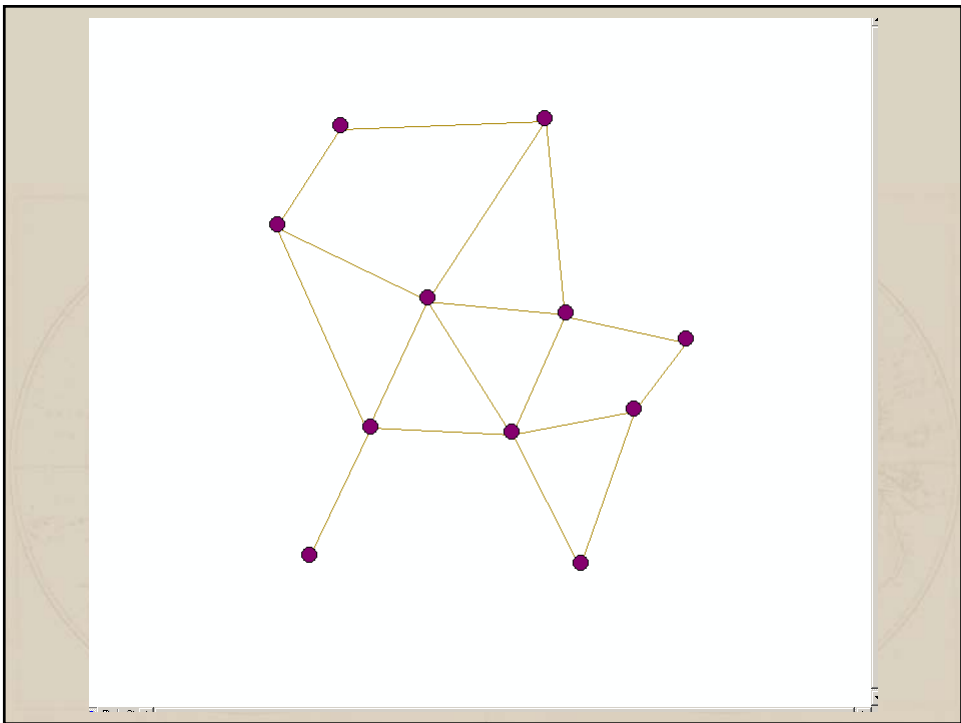
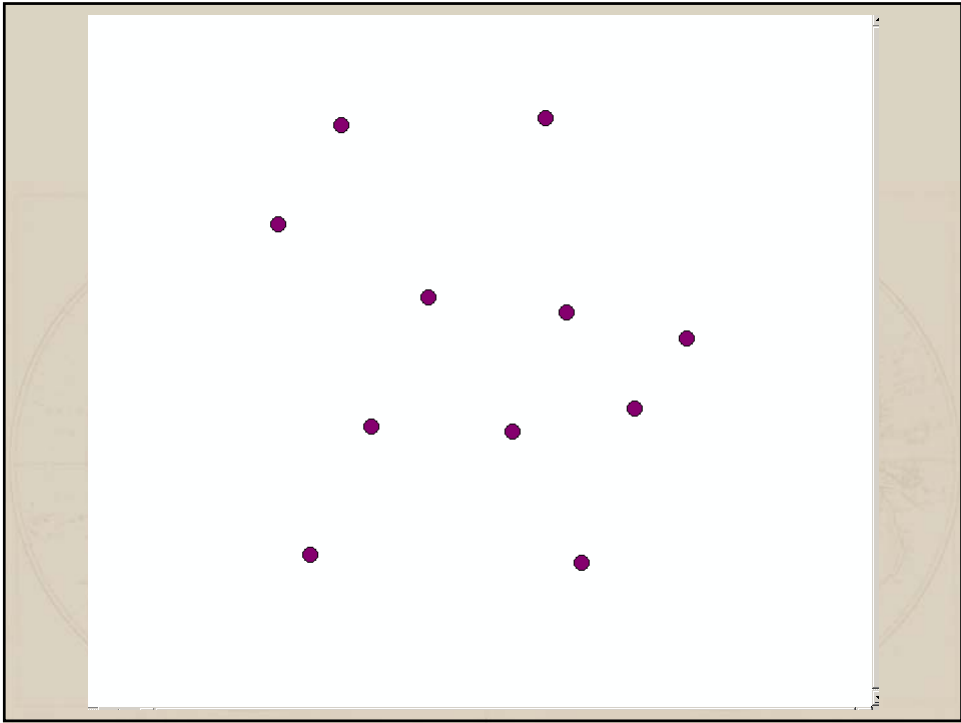


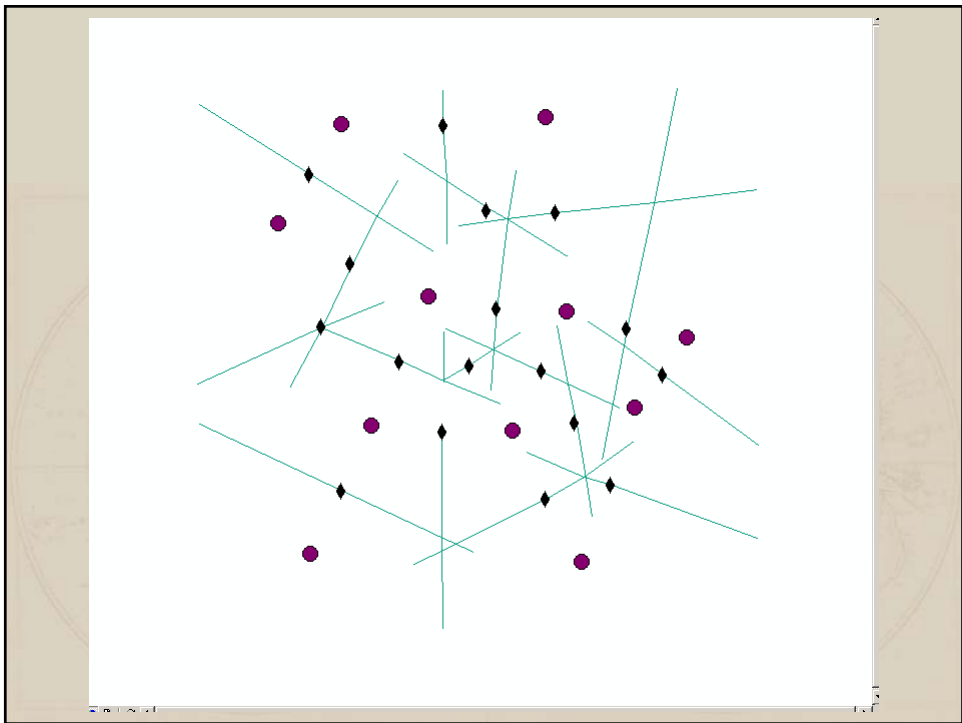
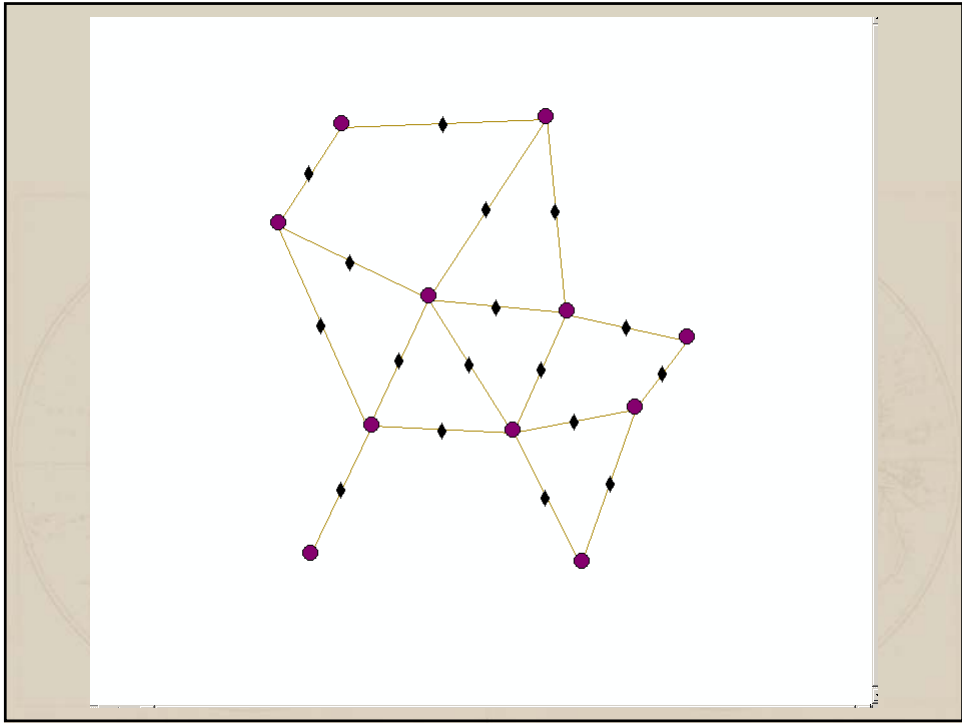
2)

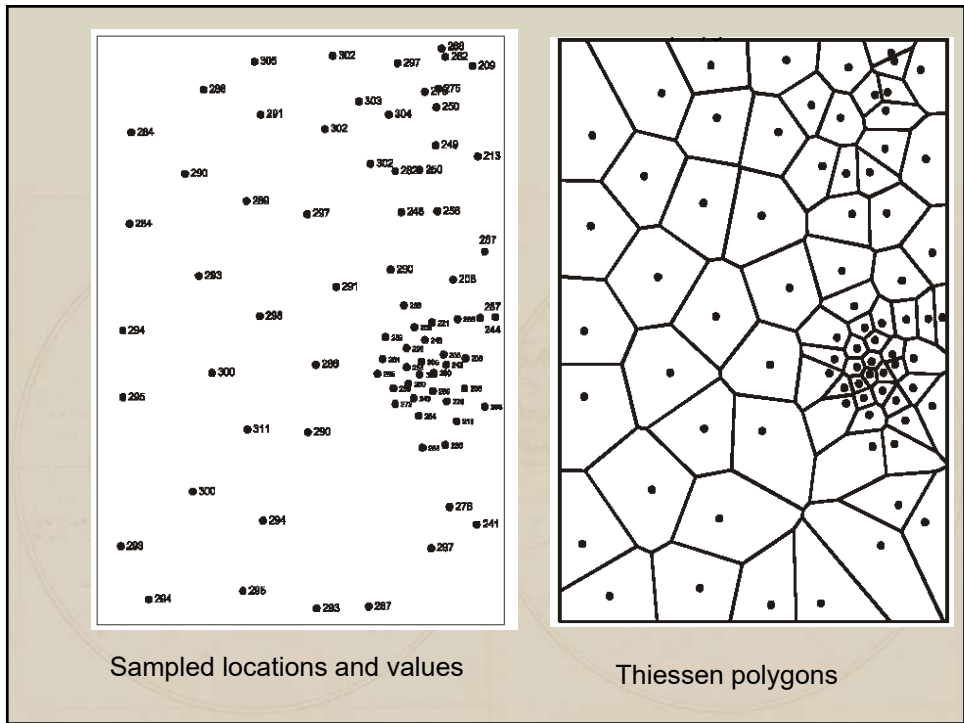
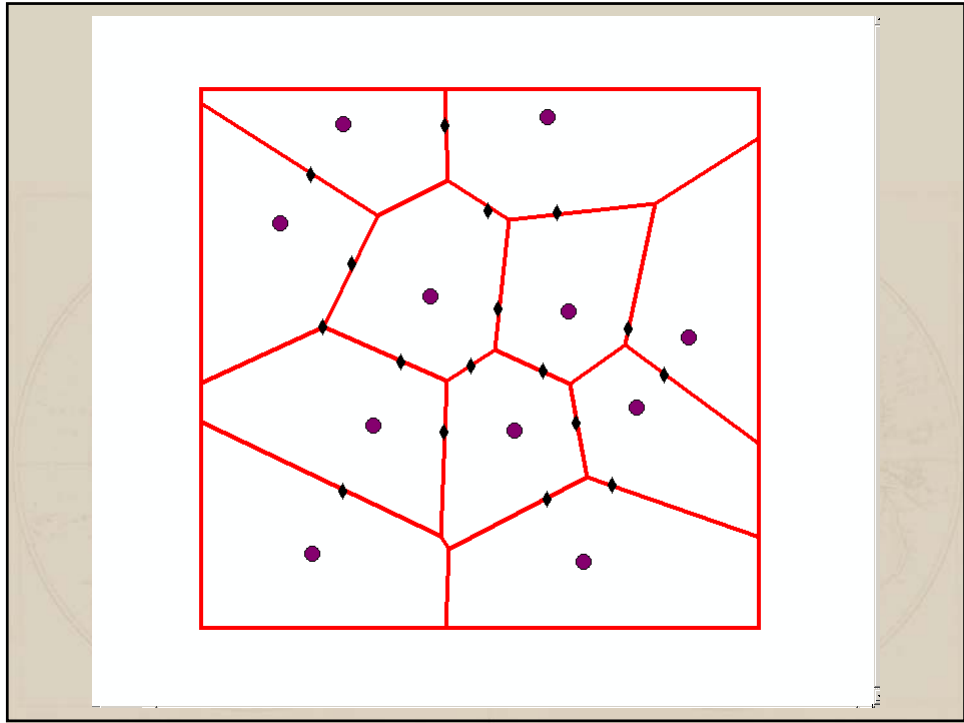


3)



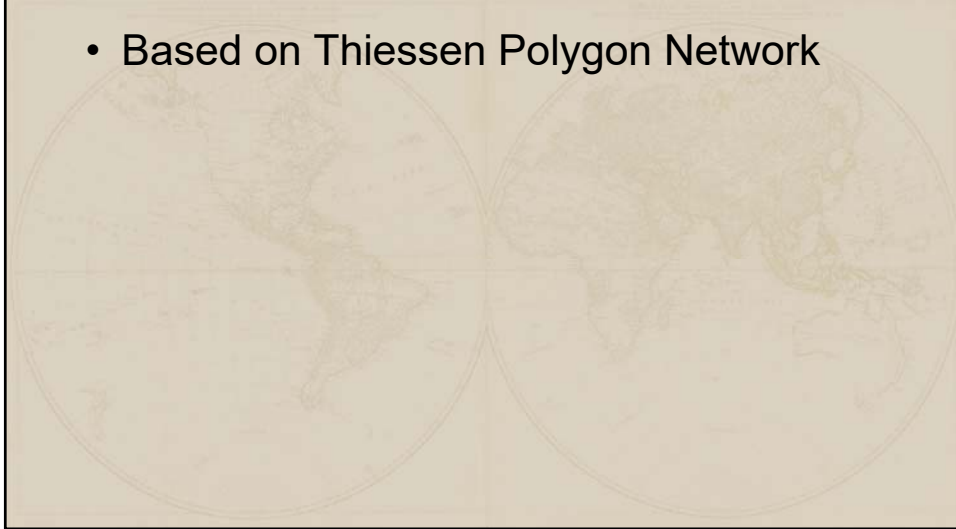






Natural neighbor interpolation

- Based on Thiessen Polygon Network



INTERPOLATION

Fixed-Radius – Local Averaging

More complex than nearest sample

Cell values estimated based on the average of nearby samples

Samples used depend on search radius

(any sample found inside the circle is used in average, outside ignored)

- Specify output raster grid
- Fixed-radius circle is centered over a raster cell

Circle radius typically equals several raster cell widths

(causes neighboring cell values to be similar)

Several sample points used

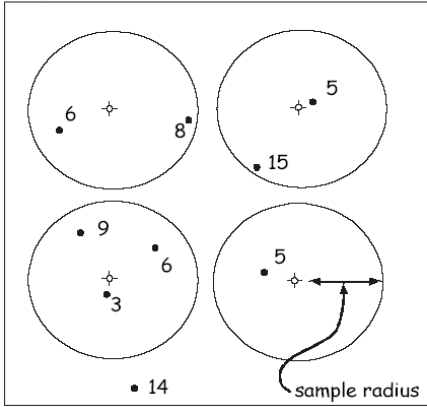
Some circles may contain no points

Search radius important; too large may smooth the data too much

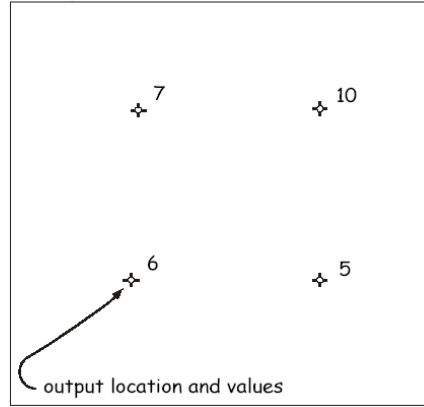
INTERPOLATION

Fixed-Radius – Local Averaging

Input

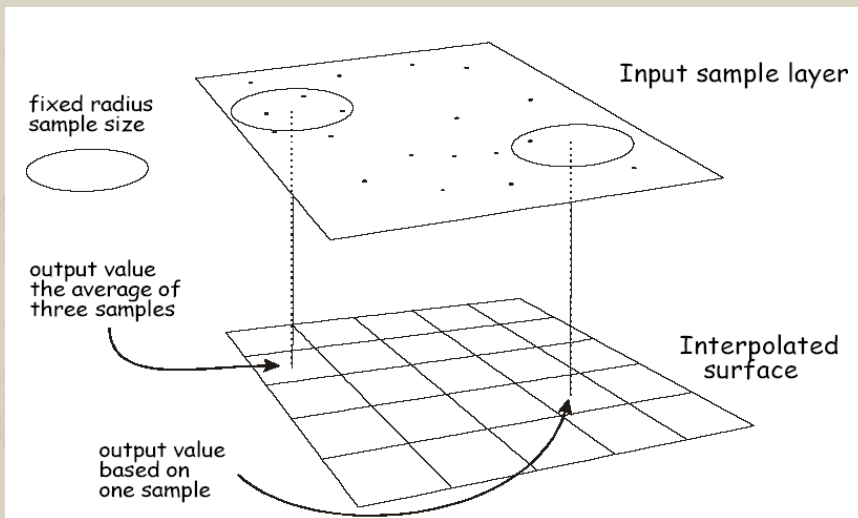


Output



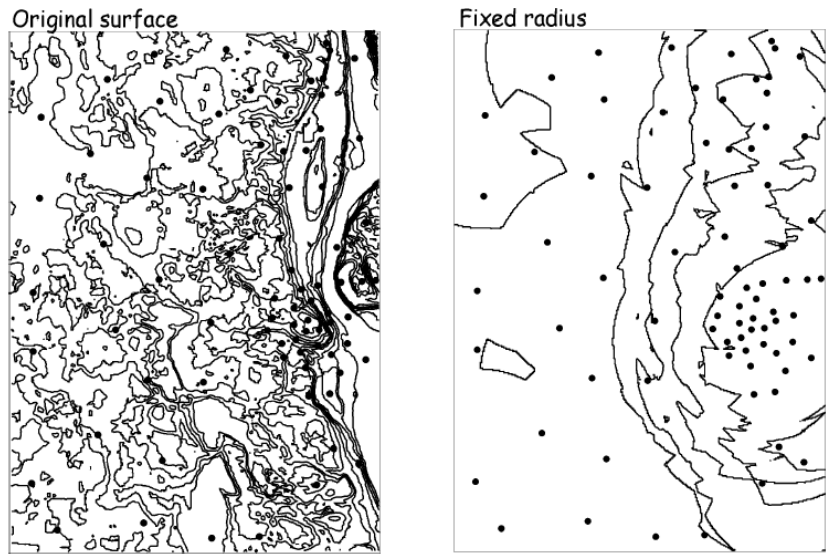
INTERPOLATION

Fixed-Radius – Local Averaging



INTERPOLATION

Fixed-Radius – Local Averaging



INTERPOLATION

Inverse Distance Weighted (IDW)

Estimates the values at unknown points using the distance and values to nearby know points (*IDW reduces the contribution of a known point to the interpolated value*)

Weight of each sample point is an inverse proportion to the distance.

The further away the point, the less the weight in helping define the unsampled location

INTERPOLATION

Inverse Distance Weighted (IDW)

Z_i is value of known point

D_{ij} is distance to known point

Z_j is the unknown point

n is a user selected exponent

$$Z_j = \frac{\sum_i \frac{Z_i}{d_{ij}^n}}{\sum_i \frac{1}{d_{ij}^n}}$$

INTERPOLATION

Inverse Distance Weighted (IDW)

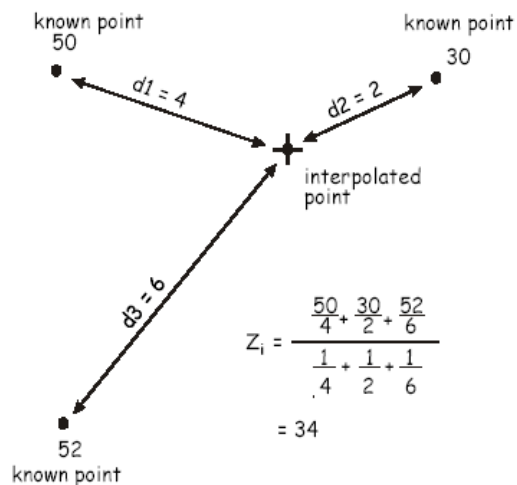


Figure 9-7: An example calculation for a linear inverse distance weighted interpolator. Known points are weighted by the inverse of their distance from the interpolation point, and summed. The sum is then divided by the sum of the weights, resulting in the interpolated value.

INTERPOLATION

Inverse Distance Weighted (IDW)

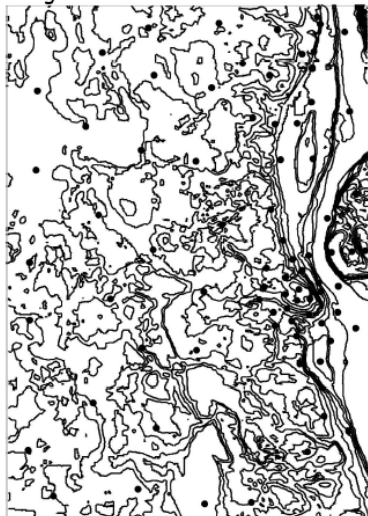
Factors affecting interpolated surface:

- Size of exponent, n affects the shape of the surface
larger n means the closer points are more influential
- A larger number of sample points results in a smoother surface

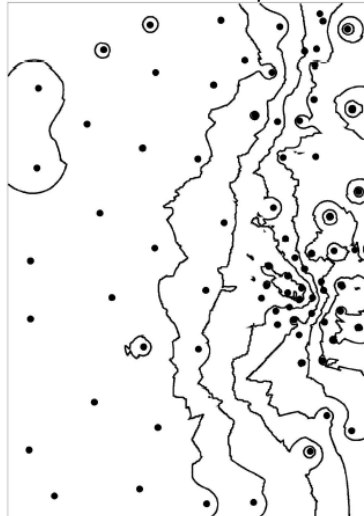
INTERPOLATION

Inverse Distance Weighted (IDW)

Original surface



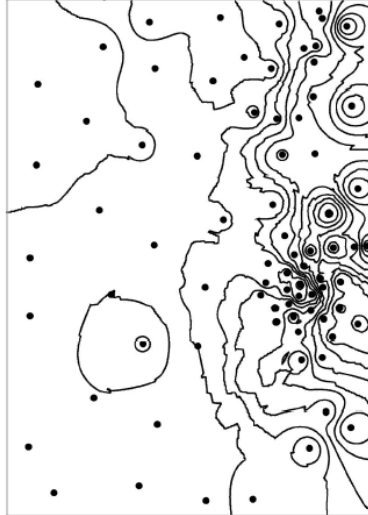
IDW - linear, 6 nearest points



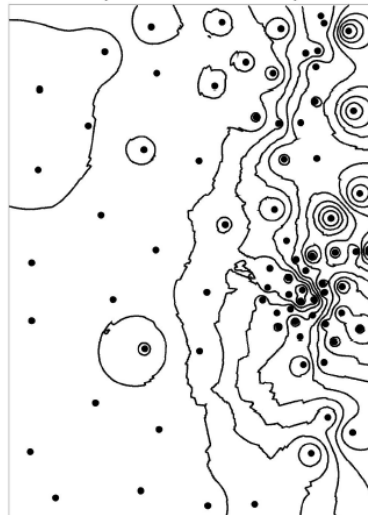
INTERPOLATION

Inverse Distance Weighted (IDW)

IDW - squared, 6 nearest points



IDW - squared, 12 nearest points



INTERPOLATION

Trend Surface Interpolation

Fitting a statistical model, a trend surface, through the measured points. *(typically polynomial)*

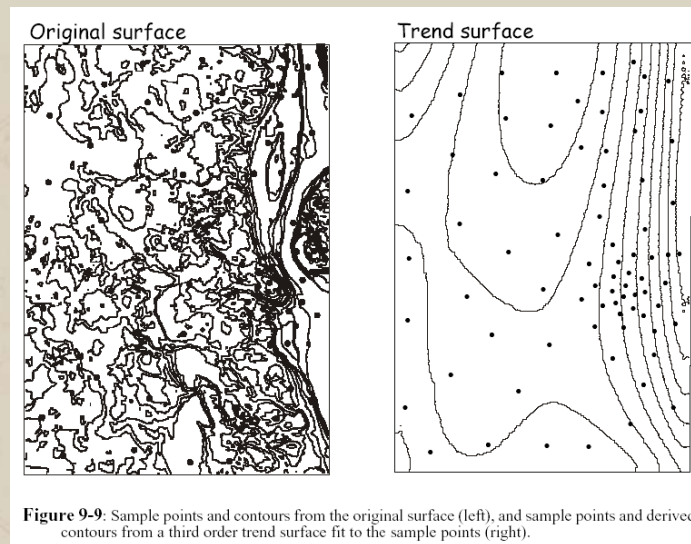
$$Z = a_0 + a_1x + a_2y + a_3x^2 + a_4y^2 + a_5xy$$

Where Z is the value at any point x

Where a_s are coefficients estimated in a regression model

INTERPOLATION

Trend Surface Interpolation



INTERPOLATION

Splines

Name derived from the drafting tool, a flexible ruler, that helps create smooth curves through several points

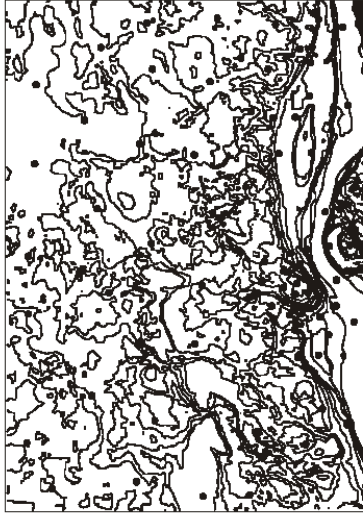
Spline functions are used to interpolate along a smooth curve.

Force a smooth line to pass through a desired set of points

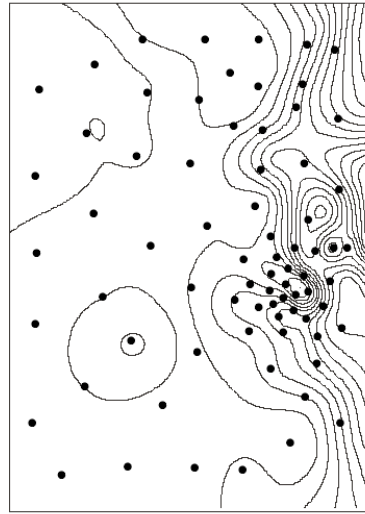
Constructed from a set of joined polynomial functions

INTERPOLATION : Splines

Original surface



Spline interpolation



INTERPOLATION

Kriging

Similar to Inverse Distance Weighting (IDW)

Kriging uses the minimum variance method to calculate the weights rather than applying an arbitrary or less precise weighting scheme

Interpolation Kriging

Method relies on spatial autocorrelation

Higher autocorrelations, points near each other are alike.

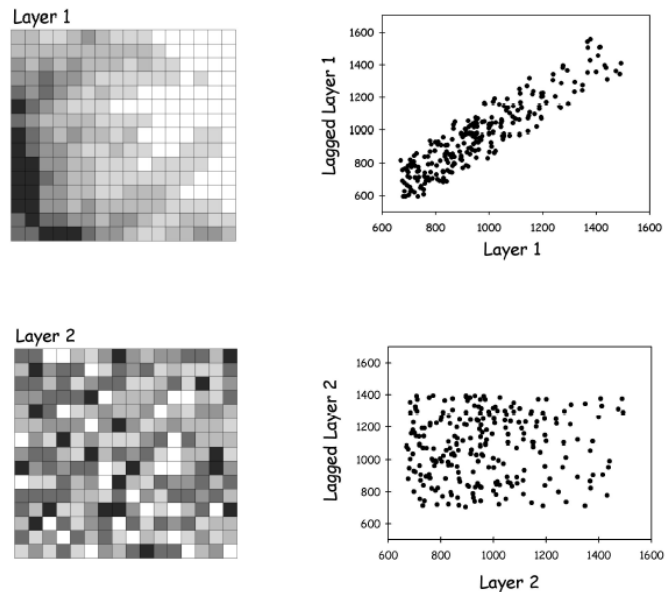


Figure 9-13: Spatially autocorrelated (top) and spatially uncorrelated (bottom) data layers. Plots of nearby sample pairs (right panels) show similar values for autocorrelated Layer 1, and un-related values for uncorrelated layer 2.

INTERPOLATION (cont.)

Exact/Non Exact methods

Exact – predicted values equal observed

Theissen

IDW

Spline

Non Exact-predicted values might not equal observed

Fixed-Radius

Trend surface

Interpolation in ArcGIS: Spatial Analyst

